SONIC FATIGUE AND RMS STRESS IN ALUMINUM STIFFENED PANELS SUBJECTED TO AN ACOUSTIC LOADING

Daniel Monteiro Eugênio, danielme73@gmail.com
Carlos Eduardo Chaves, cechaves@uol.com.br
ITA – Instituto Tecnológico de Aeronáutica

Abstract. *The purpose of this paper is to present a brief approach to the Sonic Fatigue issue on jet aircraft structures, as well as the stress applied in aluminum stiffened skin panels subjected to random accoustic loading. It will also suggest a simple and objective way to automatically estimate RMS Stress due to an acoustic loading applying methods commonly used in aerospace industry and by the meaning of easily accessible and disseminated computational tools.*

Keywords*: Sonic Fatigue, Acoustic Fatigue, RMS Stress*

1. INTRODUCTION

During the nineteen fifties, with the advance of jet engines, reports in which damages to aircraft structures close to high intensity jet streams were observed. Structural failures such as skin cracks and small cleats regularly occurred on the rear fuselage. It was noticed that the rate of failures increased when engine thrust was increased and was drastically decreased when a convergent divergent nozzle was fitted. Thus, these events were quite firmly related to jet noise. In the beginning they were considered minor damages, but were faced as a warning by aerospace industry and research centers to the possibility of further problems as the performance of aircraft and pressure ratios of engines increased. Although the majority of these failures led only to inconveniencies and costs related to maintenance rather then impacting on the aircraft life, all these cases were happening at the shadows of the event with the Havilland Comet, in Europe in January 1954, and the fear of a catastrophe was in the mind of engineers and designers.

Theoretical work and tests on simple structures such as flat and curved plates were begun. Several aircraft manufacturers and research centers set up comprehensive tests on large parts of aircraft structure in order to reproduce the sonic fatigue behavior in regions subjected to high noise intensity. During the late nineteen fifties and early nineteen sixties (decade known as the golden age of commercial aviation), jet engines were generally used on full scale models to provide realistic acoustic excitation. Measurements of the pressures in the near field region of jet exhaust were made together with strain measurements on the structure. During these tests, observations of fatigue crack initiation and endurance were recorded.

In parallel with the experience that had being building up in test programs, other theoretical studies were initiated. As new engines were developed, jet noise studies, which aimed primarily at jet noise reduction, were extended into the near field of the jet exhaust to give an increased understanding of the pressure fluctuation on the structure. Simple theoretical modes of the structure and the excitation sources were published by several investigators, between them, the seminal paper of John Miles (1954), from the United States, that gave the result of the response in one mode as well as going to consider fatigue aspects; The publication of the British Alan Powell (1958), which provided the development of normal mode formulation for linear response; And some years later, Brian Clarkson's (1968) report, with a detailed calculation of the stress on skin panels induced by a random acoustic loading, based on the single mode response.

Although the power of engines was still increasing dramatically, pressure levels were not increasing due to the use of higher bypass ratio configurations, needed to reduce noise emission. Moreover, an extensive series of studies sponsored by the US Air Force (USAF), AGARD (Advisory Group for Aerospace Research and Development) and ESDU (in that time still the UK's Engineering Science Data Unit organization) were directed toward to the establishment of a broad base of general design information applicable to varied types of structural configurations. Aerospace industry found these design guides and data sheets to be of considerable value and could, with amount of individual judgment and experience, effectively cover most commonly structure problems related to the stress induced by an acoustic loading on conventional subsonic aircraft.

In the following sections, a brief approach to the Sonic Fatigue issue on jet aircraft structures will be presented. It will be also suggested a simple and objective way to automatically estimate the RMS stress on aluminum stiffened panel subjected to a random acoustic loading, based on the "good practices guide" of ESDU reports, and using easily accessible and disseminated computational tolls, such as the Microsoft Excel.

2. SONIC FATIGUE DESIGN DETAILS AND REQUIREMENTS

According to what had been stated in the introduction section of this manuscript, the development of acoustic fatigue design criteria has historically followed on empirical approach since many of the factors that affect this type of high cyclic fatigue are not predictable. Basically, failure occurs due to pressure fluctuations in the turbulent boundary layer of a turbojet or turbofan engine exhaust stream incident on a fuselage or control surface panel, where severe noise, high temperatures, static loading and vibratory buffet occur simultaneously. Hence, Sonic Fatigue study involves many types of expertise, including aerodynamic, acoustic, vibration and metallurgic engineering.

Regarding acoustic science, tests with a single panel exhibited a response pattern which was dominated by the fundamental mode of vibration. The edge stresses were highest in this mode and fatigue failure usually began at rivet lines along the attachment between skin, stringers, frames and ribs. The effect of surrounding structure was studied by Lin (1960, 1962) theoretically and Clarkson experimentally (1962), and is likely to add more modes to the response in between the frequencies which would have been calculated for the single plate. Because of the non-uniformity of panel sizes at the rear fuselage of an aircraft (where the structure is narrowing) several low order modes are excited, but one panel will have much greater response than its neighbors. This effect is reduced as the panel sizes become more uniform. The same behavior is observed on the response of the skins and ribs of horizontal and vertical stabilizers and control surfaces.

Design requirements regarding Sonic Fatigue were issued by the FAA (Federal Aviation Administration) on 1966 by the amendment 25-10 which has introduced the fatigue evaluation to the FAR (Federal Aviation Regulations) Subpart C, Part 25.571. As a matter of fact, the paragraph "d" of the regulation number 25.571 brings the design statements just as described below:

(d) Sonic fatigue strength. It must be shown by analysis, supported by test evidence, or by the service history of airplanes of similar structural design and sonic excitation environment, that:
(1) Sonic fatigue cracks are not probable in any part of flight structure subjected to sonic excitation, or
(2) Catastrophic failure caused by sonic cracks is not possible assuming that the loads prescribed in paragraph (b) of this section are applied to all areas affected by those cracks.

The paragraph "b", mentioned above refers to damage-tolerance evaluation due to fatigue, corrosion or accidental damage.

Since then, published works related to the sonic fatigue phenomenon and especially methods of estimating the RMS (Root Mean Square) stress (S_{rms}) due to an acoustic loading have gained particular importance to aircraft manufacturers throughout the world. Due to the reasonable complexity inherent to this process, which still requires empirical input data, a program that enables RMS stress and sonic fatigue life evaluation automatically may play a great value to designers, mainly during early development or concept phase of an aircraft project.

3. RMS STRESS IN STIFFENED SKIN PANELS SUBJECTED TO RANDOM ACCOUSTIC LOADING

The high frequency response of structures to wide band random acoustic loading means that the material will be subjected to a large number of strain reversals in a normal aircraft life. Due to scatter in the results inherent to all fatigue tests, the policy adopted has been to separate out the stress estimation process from the fatigue life (N) estimation. Thus, stress prediction procedures have been used to estimate the acoustic stress (S_{rms}) at the fatigue critical points of the structure, and then, fitted on curves of S_{rms} plotted against N, previously obtained by the meaning of tests results grouped to the type of structural element, material, jointing compound and method of attachment.

As mentioned in the introduction section of this paper, the graphic procedure method provided by ESDU will be now referred and a computational program using VBA (Visual Basic for Applications) scripting language of the Microsoft Excel will be worked out in order to estimate the RMS Stress in aluminum stiffened plates subjected to random acoustic loading. This procedure basically comprises the following steps:

Estimation of fundamental natural frequency of the panel;
Obtainment of the value of spectrum level of acoustic pressure at calculated frequency.
Estimation of the RMS stress and panel's parameters calculation.

3.1. Estimation of fundamental natural frequecies of the panel

The fundamental natural frequencies of the panel, which is assumed to be rectangular, manufactured by an isotropic or orthotropic material and under static in-plane loading, will be calculating by the use of a graphic procedure provided by the ESDU report number 75030 (1975).

First of all, the edge conditions of the referred panel must be established. Data are provided by ESDU 75030 for all combination of free, simply-supported and clamped edge conditions. Boundary conditions are identified by the combination of four letters, being the first pair of letters the edge conditions of plate's longer side and the second pair the edge condition of plate's short side. The sonic fatigue phenomenon study of this paper considers conventional structure concept in which the edge condition is assumed to be the simply supported edge at all boundary of the panel, limited by stringers and frames in case of the fuselage, or by stringers, ribs or spars for stabilizers, wings, flaps and control surface panels.

Using the representation described above, the edge condition of a panel considered in this manuscript is identified by "SS-SS" (Simply Supported – Simply Supported), where its longer side is parallel to the x-axis and has its dimension represented by the letter "*a*", and the y-axis is parallel to panel's smaller side, which dimension is represented by the letter "*b*". Refer to Fig. 1 for an illustration.

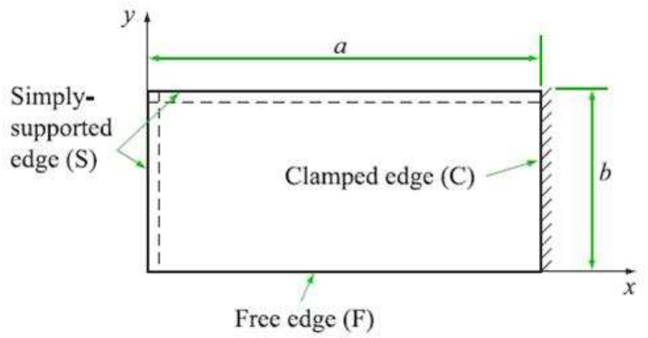

Figure 1. Sketch of panel's edge condition, orientation and dimension representation

For any mode of vibration, mode numbers "*m*" and "*n*" are related to the nodal lines provided that these lines are parallel, or approximately parallel, to the plate sides. For these cases, "*m*" and "*n*" will be one plus the number of nodal lines crossing any line parallel to the x-axis and y-axis, respectively. In this definition plate edges, supported or free, are not counted as nodal lines. Finally, considering the panel's edge condition and the vibration modes induced by an acoustic loading, the fundamental frequencies are presented in terms of the frequency coefficient "F_b", as indicates the Eq. (1) bellow.

$$f = F_b c \frac{t}{b^2} \tag{1}$$

In the Eq. (1) "*t*" is the plate thickness. The letter "*c*" is attributed to the propagation velocity of the sound wave in a given environment. For a solid, it essentially depends on the density of the material and its stress state. For a solid of density "*ρ*" and Young's module (or elasticity modulus) "*E*", the propagation velocity of a sound wave is represented by the Eq. (2).

$$c = \left(\frac{E}{\rho}\right)^2 \tag{2}$$

ESDU provides the frequency coefficient plotted against panel's aspect ratio "*b/a*" for several modes of vibration. So, if the equations of curves of "*Fb*" could be estimated, then the fundamental frequencies would be easily calculated by the use of the Microsoft Excel.

At this stage a special challenge in programming an algorithm to automatically calculate the RMS stress using the graphic method of ESDU arises. Basically, starting from an equation, the Microsoft Excel can easily plot a curve. However, how can we get an equation of an already existing curve? Or, in other words, how can we input to the Microsoft Excel data of a picture? This specific problem has been solved by the author using a program called "GetPoints".

GetPoints was developed in 2008 by the Physics Maurycy Ornaty, also Computer Engineering PhD student at AGH University of Science and Technology of Poland. It is free available on line, uses JAVA programming language and was created for simple data reading from plots commonly included in scientific papers. It is capable of capture digital images of any type and processes them according to a previous defined reference.

Using the GetPoints, data from the ESDU graphics can be exported to the Microsoft Excel, with which tendency lines can provide the equations of curves necessary to fundamental natural frequencies calculation. A result of the whole process describe above is exemplified in the Fig. 2 below.

Although the fidelity of the curves of Fig. 2 comparing with the graphic provided by the ESDU is noticeable, it is worthy to proceed with a validation of the results obtained by the use of the GetPoints. It can be done, for example, using a reverse engineering procedure in which a known equation is processed by the Excel to create a graphic. A figure of this graphic is then inputted in the GetPoints, with which a new curve is created and exported back to the Excel. Therefore, same points can be calculated using the original equation and the equation of the curve obtained from GetPoints, and the error between these two results can be finally evaluated.

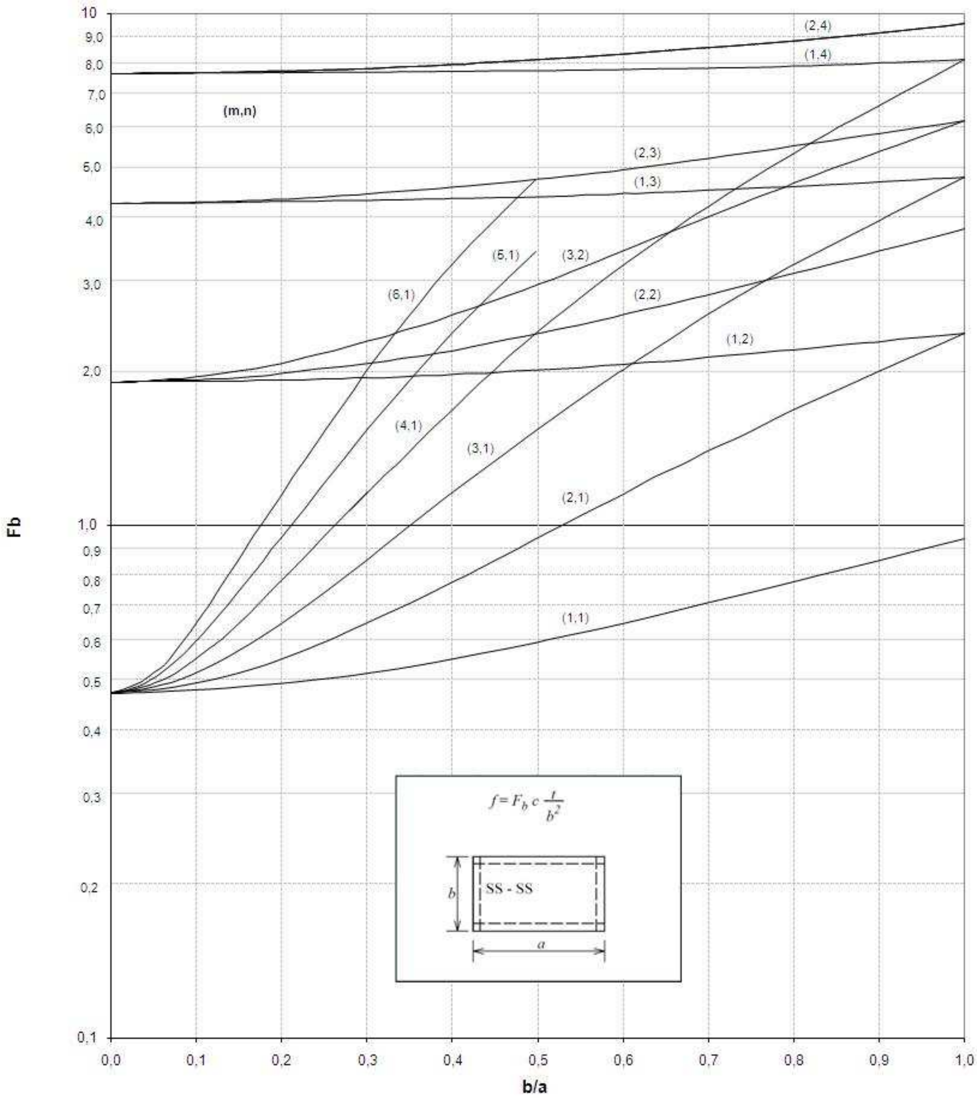

Figure 2. Result of the GetPoints for the frequency coefficient (F_b)

3.2. Computation of the spectrum level of acoust pressure at a calculated frequency

The calculation of the Spectrum Level of Acoustic Pressure, or $L_{ps}(f)$, for each one of the frequencies calculated above depends on empirical data of Sound Pressure Levels (SPL) to which panels are submitted. Aircraft manufacturers usually perform flight test in order to record SPL data, with the aid of specials microphones positioned at points of aircraft structure where sonic fatigue failure is probably to occur. A mid size commercial jet can have upon to fifty microphones installed on its fuselage, wings, stabilizers, flaps and flight control surfaces to capture SPL data while different steps of a typical flight profile are performed.

Because of characteristics inherent to the human ear, sound pressure levels data are acquired in 1/3 octave bands. However, at fundamental frequencies they must be established in 1 octave bands. The ESDU 66016 (1966) provides data for such bandwidth correction by the meaning of two graphics; One of it provides the relation between pressure levels the spectrum level at a particular frequency (in this case the frequencies previously calculated in the sub-item 2.1.) and the level in a band of known width centered on that frequency (1 octave or 1/3 octave bands); The other graphic is more generic and presents the variation of levels plotted against the respective variation of frequency for ordinary bandwidths. Nevertheless, the ESDU 66016 report also brings a numerical relation between sound pressure level and frequency for ordinary bandwidths, so user can choose either using the graphic process with the GetPoints or strength applying the Eq. (3) below. Both methods will lead to the same result.

$$\Delta L = 10 \log_{10} \Delta f \tag{3}$$

3.3. Estimation of the RMS stress and panel's parameters calculation

In accordance with ESDU 72005 (1972), the estimation of the RMS stress subjected to an acoustic loading is accomplished graphically. Starting from the spectrum level of acoustic pressure, $L_{ps}(f)$, plotted at the lower x-axis, four set of curves are presented to a given range of four different parameters. It can be said that these parameters characterize the panel because they are directly dependent on its geometry and material, including panel's properties as its aspect ratio, thickness, curvature radius and a velocity parameter (related to the sound wave propagation velocity in plate's material). Each group of curves fills up a different quadrant of the graphic.

Following the same process adopted for the frequencies determination, the GetPoints can be applied in order to obtain the equation of all the curves. However, this is not a simple case in which a known value at the abscissa hits the curve to generate another value at the ordinate. In fact, there is not a specific value to be obtained at the y-axis. After $L_{ps}(f)$ is calculated for the first parameter (the ratio *a/b*) it must meet the curve of the second parameter (the ratio *b/t*) and so on. The RMS stress will be presented at the upper x-axis after the forth parameter had been achieved. To come up with an automatically calculation of the RMS stress using the VBA toll of the Microsoft Excel, an algorithm may consider hypothetical values at both "*x*" and "*y*" axis of the graphic, in such way that the "*y*" value obtained by one curve will be used as a "*x*" value for the subsequent curve.

Another programming matter rises of the fact that there is a group of curves for each one of the panel's parameter (instead of a single curve for each parameter). As stated above, a specific range of values is attributed to each group. Hence, in order to create a generic program, applicable to panels of all dimensions and different metallic materials, a routine has to be developed to interpolate curves of the same parameter. Considering the quantity of curves existed on each group, it may demand a considerable mathematical work.

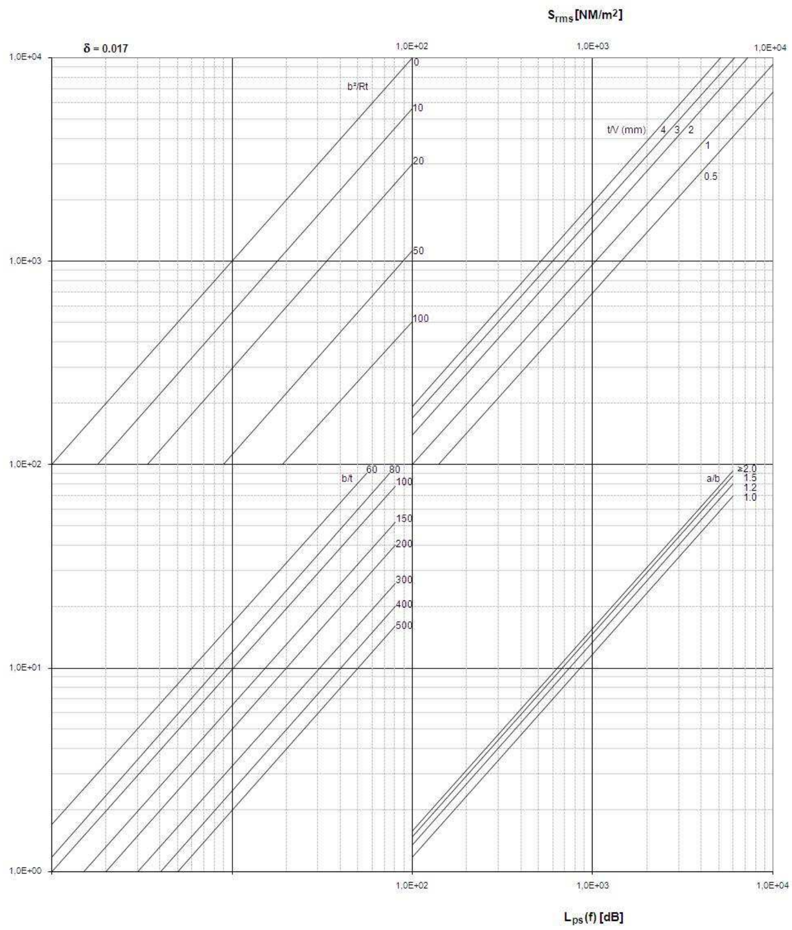

Figure 3. Result of the GetPoints for the RMS stress

Figure 3 show the first part of the procedure described above, which is the result obtained from the GetPoints inputted to the Excel. In the following item it will be presented results outputted by the algorithm developed by the author for a given panel, an analysis of these results as well as brief conclusion of this work.

4. RESULTS AND FINAL CONSIDERATIONS

The algorithm using the VBA toll of Microsoft Excel was developed to condense the calculation described on four different ESDU reports. Based on the RMS stress due to a random acoustic loading, the endurance of a similar panel manufactured by aluminum alloy is provided for different fasteners types and structural configurations, including machined reinforcements (no fasteners), one and two rows of countersunk rivets and one row of plain holes with protruding rivets. The user is required to input the dimension and material characteristics desired for the panel. The empirical sound pressure acting on the structure is also needed to be informed. Basically, the algorithm performs the following calculation:

Panel's natural frequency for several modes of vibration;
Band correction for octave bands, since the acoustic loading is obtained empirically in 1/3 octave bands;
Calculation of the sound pressure level for the given spectrum frequency;
Correction of panel's dapping ration depending on structure configuration;
Calculation of the RMS Stress acting on the panel;
The endurance estimative of a similar panel manufactured by aluminum alloy.

To exemplify the program capabilities, it will be presented results obtained by simulating an arbitrary panel subjected to an arbitrary acoustic loading. Aiming this purpose, author has chosen a typical panel of a circular fuselage with 3 meters of diameter. Skin thickness is assumed to have a standard value of 1,6 mm (approximately 0,064 inches) and sound pressure level acquired during flight tests has 135 dB intensity (typical value for maximum thrust condition – takeoff flight phase). Table 1 below presents the dimensions, parameters and characteristics of the simulated panel.

Table 1. Input values for the program

Properties of the Panel	Values	Units
Dimension "a" (bigger side)	450	mm
Dimension "b" (smaller side)	200	mm
Skin thickness "t"	1.60	mm
Fuselage Radius "R"	1.50	m
Young Module of panel's material "E"	7×10^{10}	N/m²
Density of panel's material "d"	2710	Kg/m³
Sound Pressure Level "SPL" (1)	135	dB
Single Row of Countersunk Rivets (2)	-	-

(1): Acquired in 1/3 octave frequency bands
(2): Selection of structure configuration (S-N curve database)

The results obtained by the Excel program are then provided by Tab. 2. The specific ESDU report related to each calculation step is also shown in order to provide to the reader a better link between program function and ESDU.

Table 2. Output values of the program for the 1st Mode of Vibration

Mode of Vibration	ESDU-75030	ESDU-16016	ESDU-72005	ESDU-72015
	Natural Frequency	Sound Pressure Level(1)	RMS Stress	Aluminum Endurance
(1,1)	$f_{(1,1)} = 115.6$ Hz	$L_{ps}(f) = 120.9$ dB	$S_{rms} = 5.21$ MPa	$N = 1.3 \times 10^{11}$ Cycles

(1): Band correction of Sound Pressure Level to 1 octave frequency bands

In addition, one particular output value shown on Tab. 2 can be better explored to support the program results. This validation is performed applying the Finite Element Method (FEM) to calculate the Natural Frequencies of a flat plate identical to the panel used as example (same dimensions and material properties) for several modes of vibration. The Nastran software was used to solve the eigenvalue problem of this flat plate and the result for the first mode of vibration can be seen on Fig. 4. Table 3 compares Nastran's output to the result of the Excel program. It is noticeable that the values are practically identical, and proves that the developed algorithm is in agreement with the natural frequency calculated using finite elements methodology.

Table 3. Nastran software vs. Excel program results for natural frequency of vibration

1st Mode of Vibration (1,1)	Nastran Software	Excel algorithm	Difference
	$f_1 = 115.4$ Hz	$f_{(1,1)} = 115.6$ Hz	0.17 %

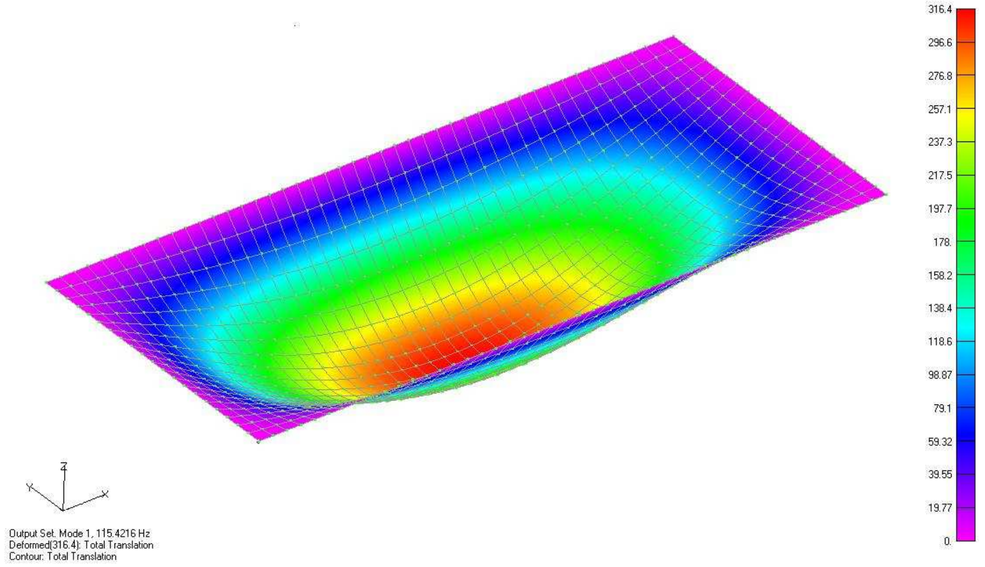

Figure 4. Nastran result for the 1st mode of a flat plate

Continuing with the results of Tab. 2, the algorithm also provides the RMS Stress used for the sonic fatigue analysis showed on Fig 5. For the adopted panel, "N_r" stands for a life of one hundred and thirty billions of cycles expected for the structure when subjected to an acoustic stress "*Srms*" of 5.21 MPa

Such endurance may seem too high compared with other fatigue lives observed on classic fracture mechanics. However, the sonic fatigue phenomenon concerns specifically when applied to structural elements for which factors that usually contribute to fatigue life decrease are not so relevant. Typical examples of these factors are pressurization cycles and high stress intensity.

In this sense, non pressurized fuselage and control surface panels could be considered less susceptive to fail due to fatigue. Nevertheless they are, coincidently, subjected to high intensity streams from jet engines; in such way that fatigue phenomenon manifests itself as a high cycle sonic fatigue due to the acoustic loading.

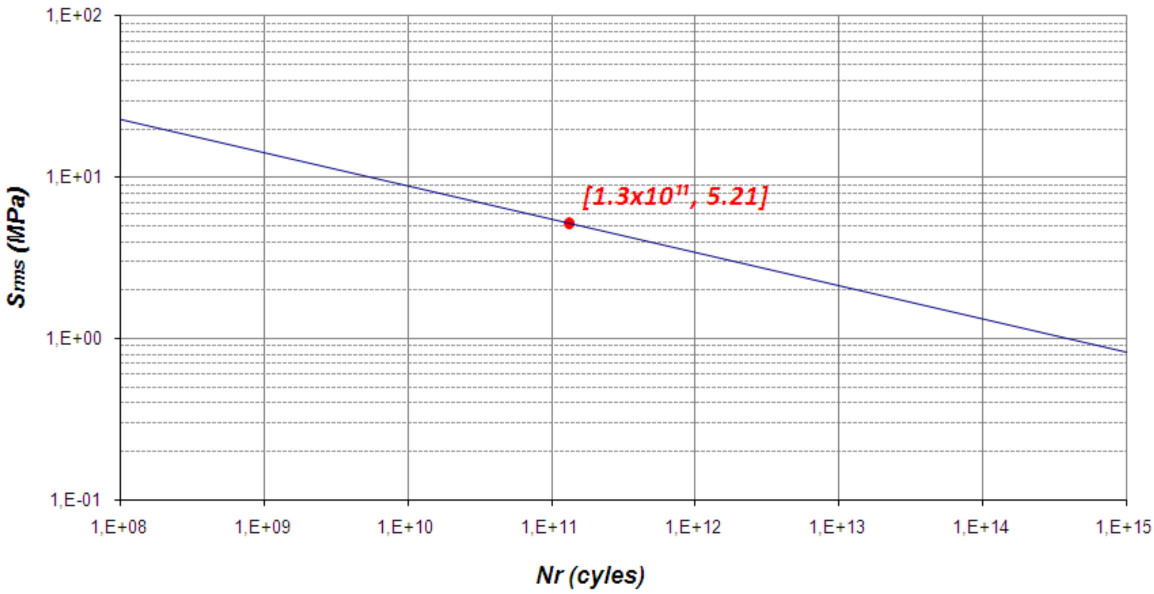

Figure 5. S-N Curve (one row countersunk rivets / skin thickness 0,064 inches)

After results have been exposed, it is realizable that the Excel algorithm developed by the author is able to replace, with pressing a single button, the calculation accomplished manually by a total of four different ESDU reports (75030, 72005, 72015 and 16016). Although it is only applicable for panels which the boundary condition is simply supported at all sides, the program definitely allows the FAA regulation number 25.571 compliance by the meaning of an automatic and easy accessible tool. And hence, sonic fatigue problem, that involves expertise as aerodynamicist, acoustician, mathematical and vibration engineering, can be faster and simply treated by structure designers and aircraft manufacturers.

5. REFERENCES

ESDU 66016, 1916, “Bandwidth corrections”

ESDU 72005, 1972, “The estimation of RMS stress in stiffened skin panels subjected to random acoustic loading

ESDU 75030, 1975, “Natural frequencies of rectangular flat plates with various edge conditions

Clarkson, B.L., 1968, “Stresses in skin panels subjected to random acoustic loads”, Journal of Royal Aeronautical Society 72.

Clarkson, B.L., 1962, “The design of structures to resist jet noise fatigue”, Journal of Royal Aeronautical Society 66.

Clarkson, B.L. and Ford, R.D., 1962, “The response of a typical aircraft structure to jet noise”, Journal of Royal Aeronautical Society 66, pp. 31-40.

Lin, Y.K., 1960, “'Free vibration of continuous skin-stringer panels”, Journal of Applied Mechanics vol. 27, pp. 669 676.

Lin, Y.K., 1962, “Stresses in Continuous skin stiffener panels under random loading”, Aerospace Sciences vol. 29, pp.67-75.

J.W. Miles, J.W., 1954, “On structural fatigue under random loading”, Journal of Acoustical Society of America, vol. 29, pp. 176-176.

Powell, A., 1958, “On the fatigue failure of structures due to pressure fields”, Journal of Acoustical Society of America, vol. 30, pp. 1130-1135.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.