Establishing Structural Repair Life by Means of Monte Carlo Simulation

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Abstract. *The purpose of this work is to incorporate the fatigue statistical scatter to the determination of fatigue life for repaired aircraft structures, by means of Monte Carlo simulation, opposed to traditional use of scatter factors. The results of the simulations indicated that even when a scatter factor of four is used for determination of a repair fatigue life, some unacceptable high probabilities of crack initiaion are still obtained. For this reason, it is recommended that a structural repair fatigue life be established based on an accumulated probability of crack initiation instead of simply using pre-defined scatter factors.*

Keywords: *repair, fatigue life, crack initiation, Monte Carlo simulation*

1. INTRODUCTION

Since the beginning of the manufacturing until the end of its service life an aircraft is subjected to numerous factors that can lead to damage or structural failure. When damage happens, the solution generally involves the application of structural repairs.

The installation of structural riveted repairs on an aircraft creates a fatigue problem that did not exist before by introducing stress concentrators (riveted holes). Through a deterministic approach based on S-N curves, Swift (1990) showed that the installation of structural repairs can significantly degrade the fatigue life of the original unrepaired structure. Swift's approach treats the initiation of fatigue cracks as a deterministic phenomenon, although it has been acknowledged the probabilistic behavior of this process, for example, as shown in Fig. 1. In practice, aircraft structural repairs are designed using the life from S-N curves divided by a safety factor that ranges from 3 to 4 as common values for metal structures, Bruhn (1973).

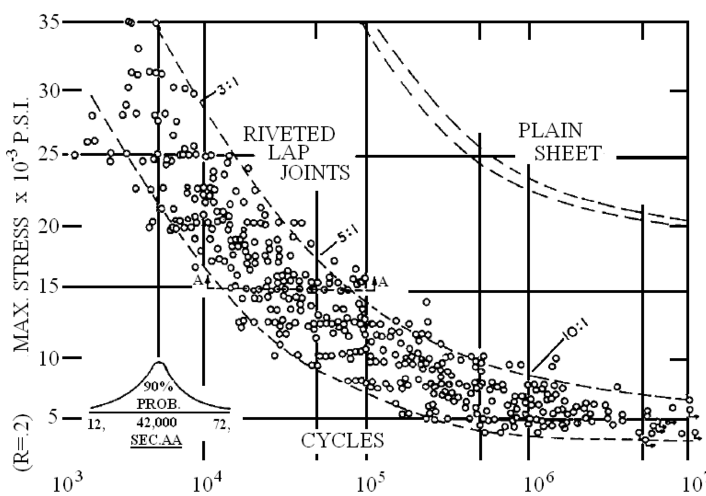

Figure 1. Statistical scattering of the S-N curve, Bruhn (1973)

Another possible way to establish a structural repair life is by extending the deterministic approach from Swift (1990) by incorporating a probabilistic analysis, via Monte Carlo simulation, to represent the scatter inherent to fatigue behavior in metals, as it will be presented in the sections to come.

2. DEGRADATION OF FATIGUE LIFE DUE TO STRUCTURAL REPAIRS

Swift (1990) demonstrated how structural repairs can degrade the fatigue life and damage tolerance capability of transport aircraft structures. Thus, when a repair is designed to restore the static strength of a structure, it should also be considered its effects on the time to crack initiation and, consequently, on the beginning of recurrent inspections of such structures.

The following example presents the traditional deterministic approach to establish a repair life, Swift (1990). Fig. 2.2 shows the reduction on fatigue crack initiation life resulting from the installation of a riveted repair (σ_{br} / σ_{gr} > 0,

Fig. 2.3) compared to an undamaged skin life subjected to a cyclic hoop stress (σ_{br} / σ_{gr} = 0, Fig. 2.1). In Fig. 2.1, the structure does not depend on loads being transferred out of the skin into the doubler as is a real repair (Fig. 2.3). The degradation of the fatigue life is a consequence of the introduction of pin-loaded holes (stress concentrators) that did not exist before.

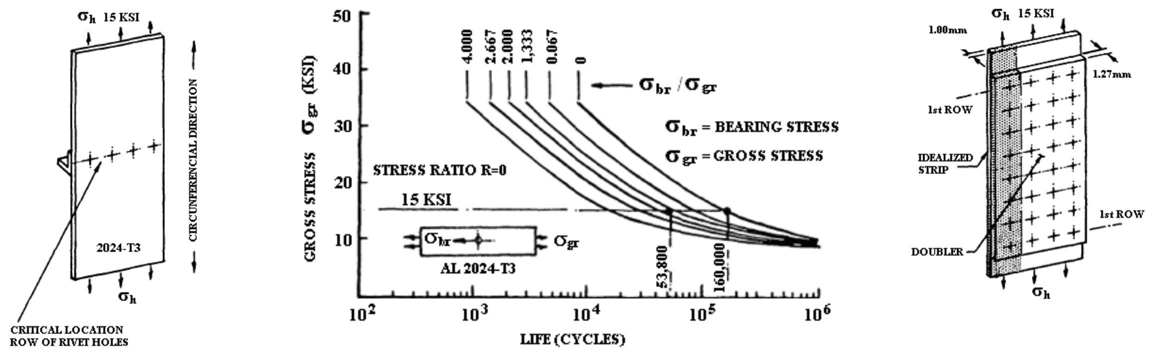

Figure 2. From left to right: 1. Undamaged skin and stringer, 2. Open hole S-N curve for Al 2024-T3, and 3. Skin with a riveted repair, Swift (1990)

The determination of fatigue life is based on the σ_{br} / σ_{gr} relationship (bearing stress/ gross stress). For example, for the skin with a typical riveted stringer (Fig. 2.1), σ_{br} / σ_{gr} = 0 leads to a fatigue life of 160,000 cycles (Fig. 2.2), while for the repaired condition (Fig. 2.3), σ_{br} / σ_{gr} = 1.642 leads to a fatigue life of 53,800 cycles (Fig. 2.2). This situation represents a considerable reduction in fatigue life (more than 66%). It should be highlighted that this approach does not consider the statistical variation inherit to fatigue behavior. S-N curves are obtained through testing specimens submitted to different stress levels to their failures.

The S-N curve is the one that best fits the experimental results for different stress levels, i.e. an average curve of 50% probability of time to crack initiation. It is expected that a deviation around this average occurs. One possibility of considering the effects of statistical variation in fatigue life obtained from S-N curves is dividing the fatigue life by a pre-defined scatter factor. Other possibility, as proposed in this work, is using a probabilistic approach by means of Monte Carlo simulation.

3. REPAIR LIFE ESTABLISHMENT METHODOLOGY

The main purpose of this work is to incorporate the statistical scatter to the determination of fatigue life for repaired aircraft structures, by using Monte Carlo simulation. An example is presented and the simulations output compared to the fatigue life obtained with the methodology described by Swift (1990), using S-N curves (50% probability), divided by a scatter factor of 4, as described by Bruhn (1973).

A simulation consists in applying computational models using mathematical techniques trying to reproduce a real event in order to understand its behavior. In this context, Monte Carlo simulation is widely used to study the time to crack initiation life, Tong (2001), as it incorporates randomness in to the study.

Equation 1 presents the function used in this work for performing the Monte Carlo simulations, Garcia (2005):

$$log(N_i) = \mu + \alpha_i \, . \, \sigma \qquad (1)$$

Where:

N_i: random fatigue crack initiation lives (in cycles) obtained from the simulations;

μ : mean life of the repaired aircraft structure (in log cycles) as calculated by Swift (1990);

α_i : random numbers from a normal distribution of mean equal to zero and standard deviation equal to 1; and,

σ: fatigue life standard deviation value (in log cycles).

As it can be seen from equation (1), the randomness of lives N_i is achieved when the random term α_i . σ is added to mean fatigue life μ obtained directly from an S-N curve. In addition to the parameters presented above, the number of holes in the 1st row of a repaired structure (also called the critical row for presenting the biggest pin-loads, Swift 1990) plays an important role on the Monte Carlo simulations. These simulations are composed of various damage scenarios. Assuming that a repair introduces *n* holes in the critical row, there are *2n* likely places where the 1st crack may start at each damage scenario: on the three and nine o'clock positions of each hole in a direction perpendicular to the resulting load. For example, in case of 5 holes in the 1st row, each damage scenario will contemplate 10 positions where cracks can start up and the simulations will calculate the time to crack initiation lives for those 10 different positions per damage scenario. The smallest fatigue life from those 10 different positions represents the first crack to initiate, and it will also be the repair fatigue life for that specific damage scenario. As the simulations go on, the number of damage scenarios increases rapidly, and a set of smallest crack fatigue lives per damage scenario is obtained. In this work, a cumulative probability distribution is obtained from this set of first cracks to initiate per damage scenario. The structural repair life is described as a cumulative distribution function where the probabilities to fatigue crack initiation is plotted against time (number of cycles).

3.1. RANDOM NUMBERS

The generation of truly random numbers is only possible through random procedures as throwing a dice. Since computers are deterministic machines and do not present random behavior it is impossible to generate truly random sequences of numbers but pseudo-random numbers, Moler (2004). This can be observed if the number generator is programmed to always use the same starting point, known as seed, the sequences of pseudo-random numbers will always be equal, Rodger (1999). Thus, in various applications, sequences of pseudo-random numbers are generated by computers. For this work, 65,000 different pseudo-random numbers α_i were generated, referred hereafter only as random numbers, which correspond to a normal distribution with mean 0 and standard deviation 1, N[0,1]. The Histogram (right) and Normal Probability Plot (left) of these numbers can be seen in Fig. 3.

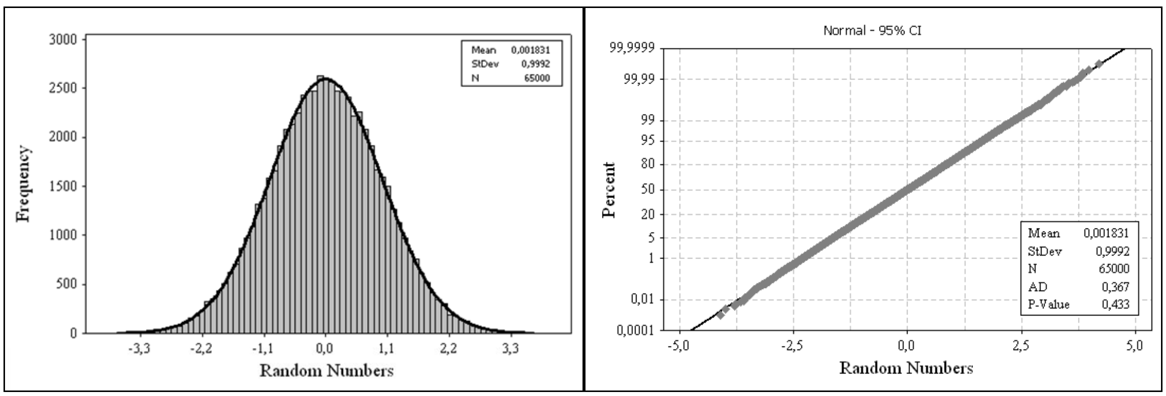

Figure 3. Histogram and Normal Probability Plot of the 65,000 random numbers from this work.

Besides the visual analysis that these graphics permit, the proximity to a normal distribution of the histogram and all the points tending to a line in the diagram of Normal Probability, the values of AD (Anderson-Darling test) and P-value greater than 5% in this last graph also shows the consistency with the fact that it was intended to demonstrate. These graphs show that the distribution of the random numbers follows the normal distribution with mean and standard deviation approximately equal to 0 and 1, respectively.

3.2. VARIATION OF MATERIAL PROPERTIES

As it can be seen from equation (1), the standard deviation value (σ), which is a material property, takes an important part on the Monte Carlo simulations. It is expected that changes on σ might considerably change the simulations results. In order to establish a range of values of σ to perform the simulations from this work, it is observed from Swift (1990) that typical structural repair lives vary from 40,000 cycles to 200,000 cycles. Many tests were performed to assess riveted aeronautical aluminum alloy joints subjected to different stress level loads and the results can be seen in Figure 4, Raikher (1999). From the work of Raikher (1999), it can be observed that for the riveted repair lives that concern this work, ranging from 40,000 cycles to 200,000 cycles (logX ranging from 4.6 to 5.3), the standard deviation values S (logX) range, approximately, from 0.01 to 0.24. Thus, as an input for the Monte Carlo simulations from this work, it will be used standard deviation values, in logarithm of cycles, of 0.01, 0.03, 0.06, 0.09, 0.12, 0.15, 0.18, 0.21 and 0 24.

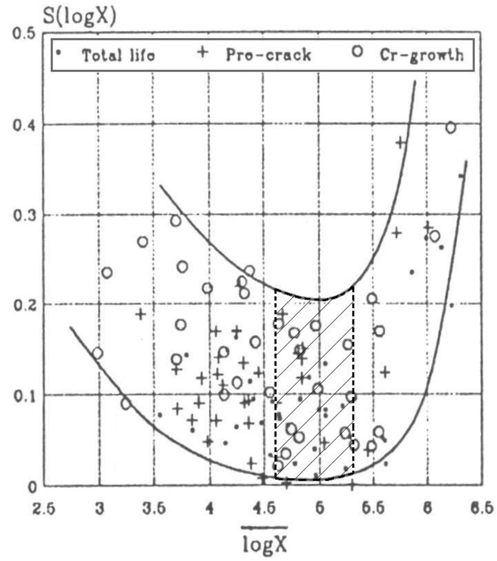

Figure 4. Relationship between mean and standard deviation values in logarithm of cycles

4. RESULTS AND DISCUSSIONS

In this section, an example on how to establish a repair life, based on a distribution of cumulative probabilities of fatigue crack initiation lives, is given. Also, the Monte Carlo simulations are performed considering 5, 10, 15, 20 and 25 holes at a repair critical row. Typical repair mean lives from $\mu = 40{,}000$ to 200,000 cycles and standard deviation values of σ = 0.01, 0.03, 0.06, 0.09, 0.12, 0.15, 0.18, 0.21 and 0.24 (equation 1) are employed. All the simulations were performed with the same number of scenarios, i.e., 1,300 scenarios each. Two examples of the simulations outcome are presented at Fig. 5 and Fig. 6.

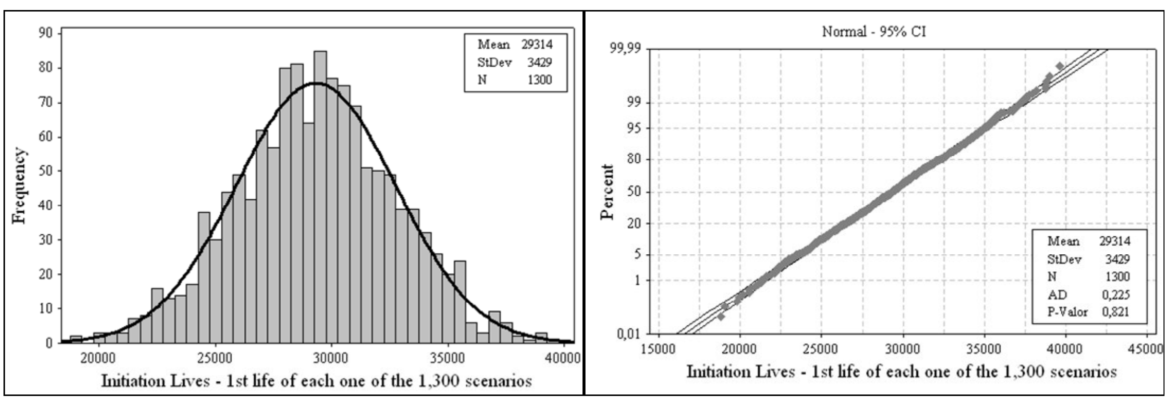

Figure 5. Histogram and Normal Probability Plot - 1st Life of Each Scenario (simulation with 5 holes, logarithm of standard deviation in log-scale of 0.09 and mean life of 40,000 cycles)

The smallest fatigue life of each scenario were plotted in a Histogram and a Normal Probability Plot, as showed in Fig. 5 and Fig. 6. The graphs show that these distributions follow the Normal Distributions.

As previously stated in this work, it is the intention of the authors to compare the establishment of a repair life based on the methodology presented in section 3 and the traditional way based on the S-N approach and the use of a scatter factor. To perform this comparison, a study case is arbitrarily selected where the mean fatigue life would be of 40,000 cycles (given from an S-N curve) and a scatter factor of 4 is employed. Traditionally, the repair life would be 40,000 cycles divided by 4, i.e., 10,000 cycles. To compare this life to the results from the Monte Carlo simulations, an example is given where $\mu = 40{,}000$ and a possible standard deviation value is arbitrarily set as $\sigma = 0.15$ (log-scale), in accordance to Fig. 4. The results from the simulations are presented in Fig. 7 as a cumulative probability distribution of lives for the initiation of the first cracks per damage scenario.

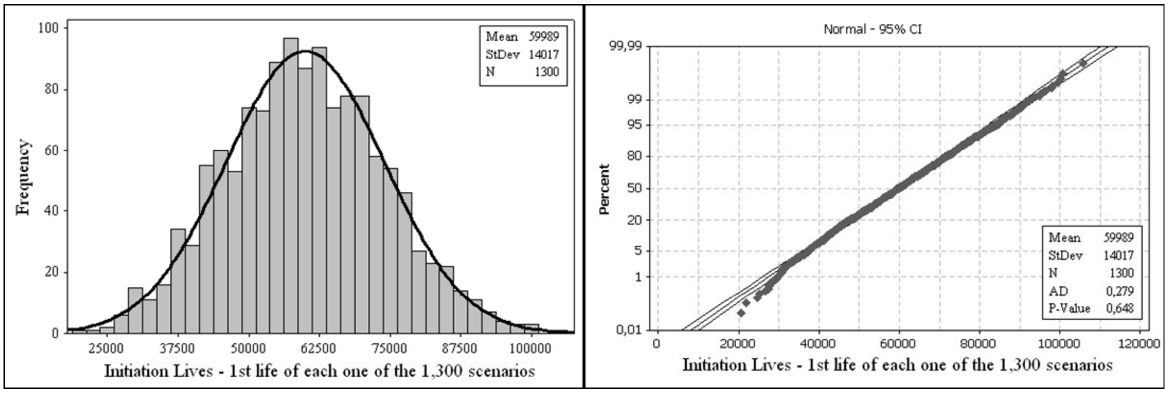

Figure 6. Histogram and Normal Probability Plot - 1st Life of Each Scenario (simulation with 25 holes, logarithm of standard deviation in log-scale of 0.24 and mean life of 200,000 cycles)

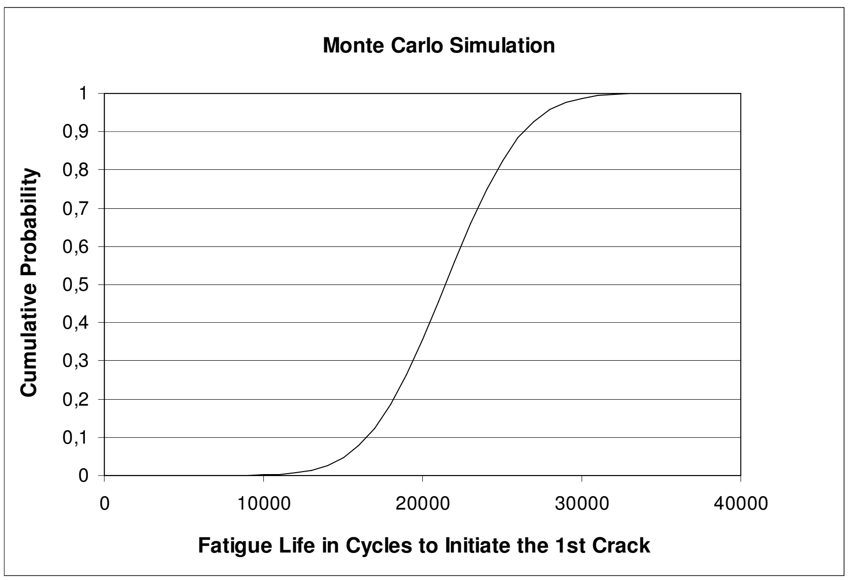

Figure 7. Cumulative Probability Distribution - 1st Life of Each Scenario (simulation with 10 holes, logarithm of standard deviation in log-scale of 0.15 and mean life of 40,000 cycles)

Before discussing the results from Fig. 7, the definition of “repair life” from this work has to be clearly established: whenever a crack initiates at a repaired structure, this moment in time is defined as the “repair life”, i.e., the repair is considered to have a failure since cracks were not supposed to take place. As it can be seen from Fig. 7, the cumulative probability distribution of lives indicate that, at 10,000 cycles, the cumulative probability to fatigue crack initiation would be of, approximately, 0.14 %. What can be realized from this probability is that it is considerably high and, therefore, unacceptable for the establishment of the repair life, even when a safety factor equal to 4 is used. According to SwRI (2007), a probability to have a failure equal to 0.14% is classified as “probable” for aircraft structures and, therefore, it should not be considered as a good reliability target. Instead of reasoning with safety factors, it seems to be much more advisable the establishment of a repair life considering a pre-defined cumulative probability to initiate a fatigue crack. To understand such procedure, the results from Fig. 7 are presented in Fig. 8, but in logarithm scale at the Y axis for better visualization. From Fig. 8, if a cumulative probability of crack initiation is set as 10^{-5} (classified as “improbable”, SwRI-2007), the repair life would be of, approximately, 5,000 cycles and not 10,000 cycles. Applying such procedure for the establishment of a repair life gives the analyst the possibility to choose to be more or less conservative, whenever he needs.

In order to complete the results from the example presented in the previous paragraph, it seems to be important to highlight that the standard deviation value used in such analysis could be ranged from 0.01 to 0.24, for the mean life of 40,000 cycles, as presented in Fig. 4. The number of holes at the repair critical row will also vary from 5 to 25. The results from these simulations are presented in Fig. 9. These results refer to the cumulative probabilities for initiation of fatigue cracks at a fixed life of 10,000 cycles. As it can be seen from Fig. 9, the probabilities of initiation of fatigue cracks begin to increase significantly for standard deviation values greater than 0.15. These results demonstrate that the knowledge of standard deviation values for the establishment of a repair life is a serious issue. It can also be seen that as the number of holes in a repair row increases, the cumulative probability for crack initiation increases, which is physically consistent as the number of fatigue critical locations are increasing.

From the results given in this section, it can be concluded that a better way to establish the fatigue life of a repaired structure is by means of a pre-defined cumulative probability for fatigue crack initiation, which is preferred to establish the same life based on the S-N approach coupled with safety factors. It could also be seen that the cumulative probabilities for the initiation of fatigue cracks change considerably as a function of the standard deviation and the number of holes.

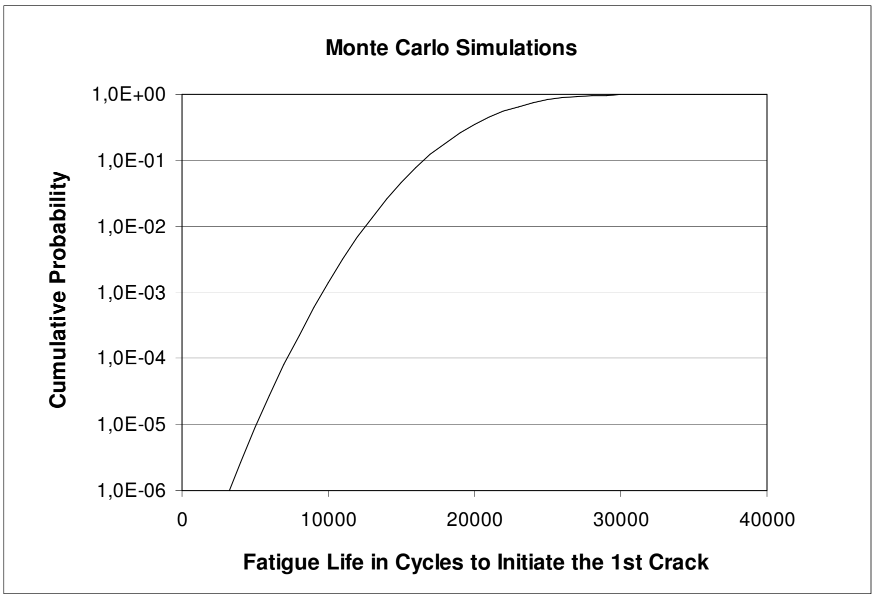

Figure 8. Cumulative probability distribution from figure 7.

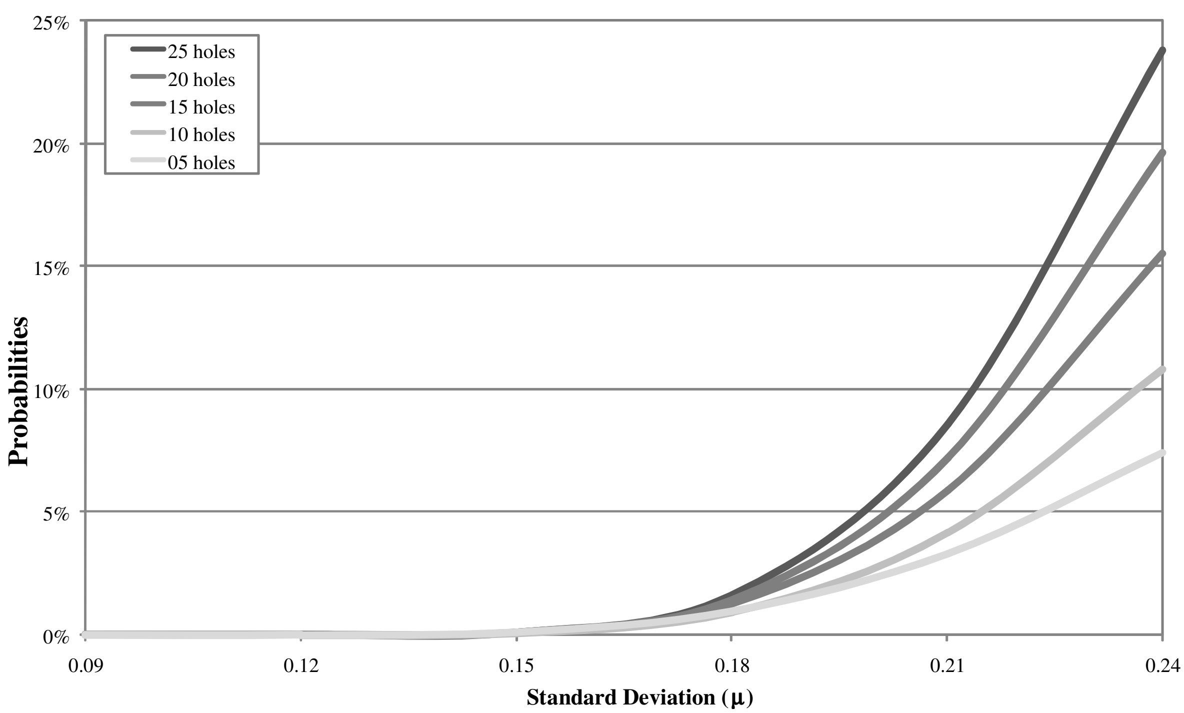

Figure 9. Crack initiation probability in riveted repairs

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6. RESPONSIBILITY NOTICE

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