

# INVESTIGATION OF MULTIVARIABLE CONTROL METHODS FOR ATTITUDE CONTROL OF AN ARTIFICIAL SATELLITE

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**Abstract.** *In this paper are investigated multivariable control methods for attitude control of an artificial satellite consisting of a rigid body and two flexible panels. The investigated techniques are Linear Quadratic Regulator (LQR) method, Linear Quadratic Gaussian (LQG) method and H-Infinity method. The satellite modeling was built following the Lagrangean approach and the discretization was done using the assumed-modes method. The equations of motion obtained were written in its modal state space form. In LQR method, the control law shows a good performance, however the method is only applied for system with absence of disturbances and where all the states are available. In reality that does not happen. On the other hand, the LQG method is more realistic, because nor all the states are available and the system presents noises. However, the performance of the system decays due to presence of the Kalman filter. The disadvantage of both the methods is the absence of a systematic procedure in the choice of weights matrix  $Q$  and  $R$ , and noises  $w$  and  $v$ . The  $H_\infty$  method has a distinct systematic with respect to the other two methods here applied. In comparison with LQR and LQG the  $H_\infty$  results in this work had been superior. However its great disadvantage is in the need of great ability and necessary experience to build the weights that are associated to the performance of the method  $H_\infty$ .*

**Keywords:**  $H_\infty$ , LQR, LQG, attitude control, artificial satellite.

## 1. INTRODUCTION

Future spacecraft will be considerably more flexible, structurally, than their current counterparts. Albeit the use of small artificial satellites has shown a relatively fast way simple and of low cost for reaching the space in space missions within the most of several applications, it is evident, that the conquest of the space will not be possible without the employment of Rigid-Flexible Satellites (RFS) in missions of larger complexity (Cubillos, 2008).

However, the control and positioning of the flexible manipulator system is more difficult than rigid one. For that reason an extensive amount of research has been done in the area of active vibration control in aerospace structures such as flexible aircraft, satellites, space antennas and more recently the International Space Station (ISS). To the control of RFS, Joshi (1989) treats in details the problem, where the main tasks for the Attitude Control System (ACS) are: i) fine-pointing of some of the appendages to different targets, ii) rotating of some of the appendages to track specified periodic scanning profiles, and iii) changing the orientation of some of the appendages through large angles.

The function of the ACS is to stabilize and orient the satellite during its mission, counteracting external disturbances torques and forces. Depending on the complexity of the satellite mission the ACS design methods can be based on linear or on nonlinear dynamics (Souza, 2006).

Considering that the most important objective of projecting a control system with feedback is to reach stability and the nominal specification of acting for a certain plant. And to keep this performance independent of errors between the project model and the real model and of the variation of the parameters of the system. Thus, one concludes that the procedure to project a control system is a difficult task due to the cited requirements are conflicting properties (SAFONOV, 1981). Moreover, nor always it is possible to include both the sources of errors simultaneously in the same procedure of project of the control system. The first is usually characterized by means of models in the domain of the frequency, and the second is represented through models in the space of states.

The wishes and needs to precisely control a spacecraft's attitude has led to active research in the attitude control throughout the years. In this paper are investigated multivariable control techniques for attitude control of a artificial satellite. The methodologies are: Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG) and H- Infinity ( $H_\infty$ ) and will be evaluate the performances in the ACS of a RFS.

## 2. THE ARTIFICIAL SATELLITE

### 2.1. Rigid and flexible model

In the Fig. 1 shows the artificial satellite model constituted of a "rigid body" of cubic form and two flexible panels. The center of mass of the satellite is in the point 0 origin of the system of coordinates (X, Y, Z), that coincides with main axis of inertia. The elastic appendixes with the beam format are connected in the central body, being treated as a

punctual mass in its free extremity. The length of the panel is represented by  $L$ ,  $m$  is mass and  $v(x, t)$  is elastic displacement in relation to the axis  $Z$ . The moments of inertia of the rigid body of the satellite in relation to the mass center it is  $J_0$ . The moment of inertia of the panel in relation to its own mass center is given by  $J_p$ .

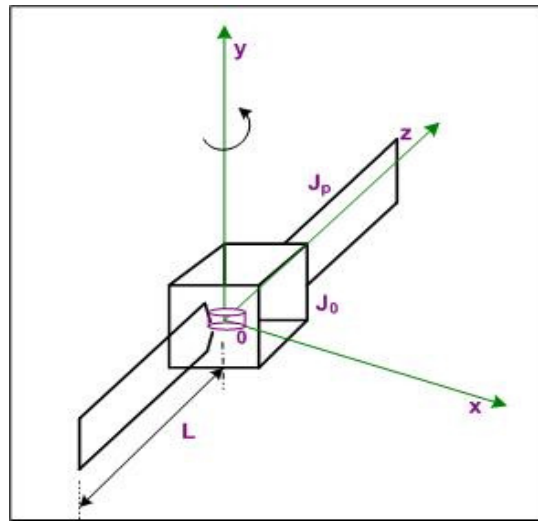


Figure 1. Artificial satellite model

## 2.2. Mathematical equations

This mathematical model is obtained using the assumed mode method and the Lagrange equation (Cubillos, 2008). The beam deflection variable  $v(x, t)$  is discretized using the expansion:

$$v(x, t) = \sum_{i=1}^n \phi_i(x) q_i(t) \quad 0 \leq x \leq L \quad (1)$$

$n$  represents the number of manners to be adopted in the discretization  $\Phi_i(x)$  it represents each one of the own modes of the system. The admissible functions  $\Phi_i(x)$  are given by (Craig, 1981) and (Junkins and Kim, 1993):

$$\phi_i(x) = \cosh(a_i x) - \cos(a_i x) - \alpha_i (\sinh(a_i x) - \sin(a_i x)) \quad (2)$$

where:

$$\alpha_i = \frac{\cosh(a_i L) + \cos(a_i L)}{\sinh(a_i L) + \sin(a_i L)} \quad (3)$$

In the Lagrange approach are considered the motion of rotation of the satellite around in  $Y$  and the elastic displacement of the panels. The equations of Lagrange, (Meirovitch, 1998), for the problem in subject are written in the following form:

$$\frac{d}{dt} \left( \frac{\partial L^*}{\partial \dot{\theta}} \right) - \frac{\partial L^*}{\partial \theta} = \tau \quad (4)$$

and

$$\frac{d}{dt} \left( \frac{\partial L^*}{\partial \dot{q}_i} \right) - \frac{\partial L^*}{\partial q_i} + \frac{\partial M}{\partial q_i} = 0 \quad (5)$$

In the Eq. (4)  $\tau$  is the torque of the reaction wheel;  $L^* = T - V$  is the lagrangean and  $\theta$  the angle of rotation of the satellite around the axis  $Y$ . In Eq. (5)  $M$  it is the dissipation energy associated to the deformation of the panel,  $q_i$  it represents each one of the generalized coordinates of the problem.

For the complete system, the total kinetic energy  $T$  is given by  $T = T_{\text{Satellite}} + T_{\text{Panel}}$ , therefore

$$T = \frac{1}{2} J_0 \dot{\theta}^2 + \left[ \rho A' \int_0^L \left[ \dot{v}(x,t)^2 + 2(\dot{v}(x,t))x\dot{\theta} + (x\dot{\theta})^2 + (\dot{\theta}\dot{v}(x,t))^2 \right] dx \right] \quad (6)$$

where  $\rho$  is the density of the panels and it is the area of the same. The dissipation energy function is

$$M = \dot{v}(x,t)^2 K_d \quad (7)$$

$K_d$  is the dissipation constant. So  $L^* = T - V$  is given by

$$L^* = \frac{1}{2} J_0 \dot{\theta}^2 + \left[ \rho A' \int_0^L \left[ \dot{v}(x,t)^2 + 2(\dot{v}(x,t))x\dot{\theta} + (x\dot{\theta})^2 + (\dot{\theta}\dot{v}(x,t))^2 \right] dx \right] - v(x,t)^2 \cdot K \quad (8)$$

In Eq. (8)  $K$  is constant elastic of the panels. After some manipulations (Cubillos, 2008), expanding  $v$  in Eq. (8) and through the property of orthogonalization of the modes of vibration of the beam (Hassmann and Fenili, 2007):

$$\int_0^L \phi_i \phi_j dx = 1 \quad \text{if} \quad i = j. \quad \int_0^L \phi_i \phi_j dx = 0 \quad \text{if} \quad i \neq j. \quad (9)$$

We have, finally, the two equations of the motion found, that represent the dynamics of the motion of rotation of the satellite and the elastic displacement of the panels, respectively:

$$\ddot{\theta} \left( 1 + a \sum_{i=1}^n q_i^2 \right) + \alpha_i a \sum_{i=1}^n \ddot{q}_i = \frac{1}{J_1} \tau \quad (10)$$

$$\ddot{q}_i + \alpha_i \ddot{\theta} - \dot{\theta}^2 q_i + d \dot{q}_i + c q_i = 0 \quad (11)$$

where the term no-linear  $\alpha_i$  in Eq. (10) is defined centripetal rigidity and the constants are:

$$a = \frac{2\rho A'}{J_1}; \quad J_1 = J_0 + 2J_p; \quad c = \frac{K}{\rho A'}; \quad d = \frac{K_d}{\rho A'}$$

### 2.3. State space form

The governing equation of motion given by Eqs. (10) and (11) are now written in state space form. From here is consider  $i=1$  (one mode). The state vector is defined as:

$$x = \{x_1, x_2, x_3, x_4\}^T \quad (12)$$

where  $x_1 = \theta, x_2 = \dot{\theta}, x_3 = q_1, x_4 = \dot{q}_1$ .

Using the vector defined in Eq. (12), the governing equations of motion in state space form are written as:

$$\dot{x}_1 = x_2 \quad (13a)$$

$$\dot{x}_2 + a\alpha_1 x_4 = \frac{1}{J_1} \tau \quad (13b)$$

$$\dot{x}_3 = x_4 \quad (14a)$$

$$\dot{x}_4 + \alpha_1 \dot{x}_2 = -d x_4 - c x_3 \quad (14b)$$

Equations (13) and (14) can be written in matrix form as:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_1.a \\ 0 & 0 & 1 & 0 \\ 0 & \alpha_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -c & -d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/J_1 \\ 0 \\ 0 \end{pmatrix} \tau \quad (15)$$

Multiplying eq. (15) on the left by the inverse of the first matrix in this same equation, one will have:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_1.a.c}{-1+\alpha_1^2.a} & -\frac{\alpha_1.a.d}{-1+\alpha_1^2.a} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{c}{-1+\alpha_1^2.a} & \frac{d}{-1+\alpha_1^2.a} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ \alpha_1 \end{pmatrix} \frac{1}{(-1+\alpha_1^2.a).J_1} \tau \quad (16)$$

### 3. THE MULTIVARIABLE CONTROL METHODS

#### 3.1. The LQR method

Given a controllable and observable system, exists a linear control law  $u$ , such that, minimizes the deterministic cost

$$J = \int_0^{\infty} (x^T(t)Qx(t) + u^T(t)Ru(t))dt \quad (17)$$

where the matrices  $Q$  and  $R$  defined semi-positive  $R$  positive defined, respectively. The dynamic of the system is represented by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (18)$$

and the control law is defined by

$$u = -K_r(t)x(t) \quad (19)$$

where  $K_r(t)$  is given by

$$K_r = R^{-1}B^T P(t) \quad (20)$$

with  $P(t)$  solution of Riccati equation

$$-\dot{P} = A^T P + PA + Q - PBR^{-1}B^T P \quad (21)$$

In the stationary case, the Riccati equation is equal to zero. The LQR technique is more appropriate for systems that possess project models reasonably exact and ideal sensor/actuators; and in the preliminary of the project of the control laws (Cubillos, 2008).

#### 3.2. The LQG method

In the Linear Quadratic Gaussian (LQG) design shown in the system is assumed to have random disturbance inputs and assumes the presence of sensor noise (Cubillos, 2008). Considers the state estimation problem of the stochastic system given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gw(t) \\ y(t) &= Cx(t) + v(t) \end{aligned} \quad (22)$$

where  $w(t)$  and  $v(t)$  are Gaussian noises with mean zero having covariances

$$\begin{aligned} E\{w(t)w^T(t)\} &= W \geq 0 \\ E\{v(t)v^T(t)\} &= V > 0 \\ E\{w(t)v^T(t)\} &= 0 \end{aligned} \quad (23)$$

The input  $u(t)$  represents the control vector and  $y(t)$  the vector of measured outputs. One refers to  $w(t)$  and  $v(t)$  as the system noise and the noise of the measures, respectively. The solution of the LQG problem consists in obtained a feedback control law that minimizes the cost

$$J = \lim E \left\{ \int_0^{\infty} (x^T(t)Qx(t) + u^T(t)Ru(t)) dt \right\} \quad (24)$$

The solution to the LQG problem is prescribed by the separation principles (Kwakernaak, 1972) which reduce the problem to two sub-problems, see the representation in Fig. 2.

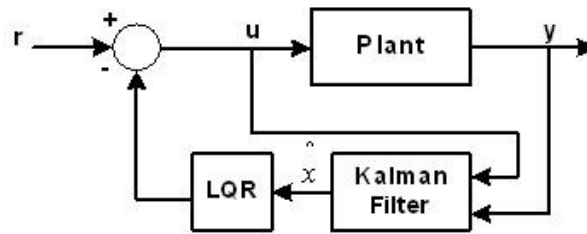


Figure 2. LQG control law

The first sub-problem is the Kalman Filter that is given by a state estimator of the form

$$\dot{\hat{x}}(t) = (A - K_f C)\hat{x}(t) + Bu(t) + K_f y \quad (25)$$

with the control law  $u = -K_r \hat{x}$  based on the estate estimated vectors  $\hat{x} = x_{est}$ . The Kalman filter gain is given by

$$K_f = P_k C^T V^{-1} \quad (26)$$

where  $P_k$  satisfies another algebraic Riccati equation

$$0 = AP_k + P_k A^T + GWG^T - P_k C^T V^{-1} C P_k \quad (27)$$

Once obtained the estimated states, passes to the second sub-problem, which is to get an optimal control law, based on the LQR method. From the design perceptive one has to find a compensator with a structure that is a series connection of a KF with a state feedback matrix. It is very well known that the optimal LQR and the Kalman filter have very good robustness and performance properties when are designed separately (Cubillos, 2008).

### 3.3. H – infinity method

The growing increase of more complex systems and processes that need to be controlled, the development of analysis methods as project of more sophisticated control systems has been providing. Introduce by Zames (1981), the  $H_\infty$  control theory combines both answers: the domain of the time and of the frequency in order to supply a unified solution. Throughout the decades of 1980 and 1990 had a significant impact in the development of control systems, nowadays the technique has been ripening and their applications in industrial problems are every time larger (Cubillos, 2008).

The advantage of using the  $H_\infty$ -method its the ability of including the solution in the equations of an optimization problem. The performance objectives are: bandwidth and the resolution of function cost. In Fig. 3 demonstrate the generic representation of system.

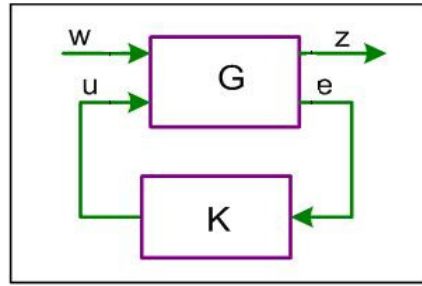


Figure 3. Augmented plant for H $\infty$  design

The sign ( $w$ ) represents the external input to the system; ( $z$ ) is the error sign; ( $u$ ) is the control sign; ( $e$ ) it is the sign of difference between the output ( $y$ ) and the input ( $w$ ). The control problem is to determine a controller  $K$  to stabilize  $G$  (augmented plant) and minimize the functions transfer between ( $w$ ) and ( $z$ ).

The widespread model of the system Eq. (28) and (29).

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{cases} \quad (28)$$

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} = P \quad (29)$$

The augmented plant is formed by accounting for the weighting functions  $W_1$ ,  $W_2$  and  $W_3$ , as show in Fig. (4). In order to reach the acting objectives, the outputs were chosen to be transfer weight functions,  $z_1 = W_1$ ;  $z_2 = W_2 y$ ;  $z_3 = W_3 u$ .

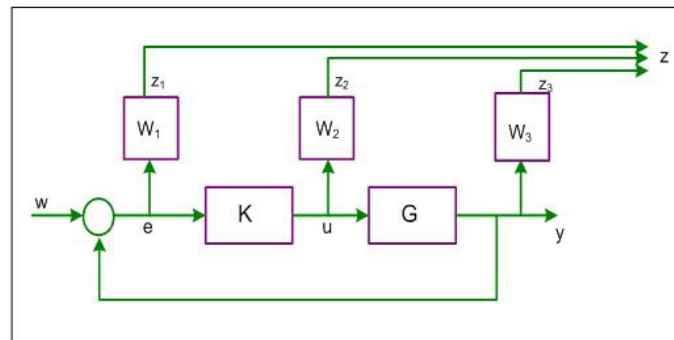


Figure 4. Plant with weighting functions for H $\infty$  design

The function cost of mixed sensibility is given for:

$$T_{y|w} = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \quad \begin{aligned} S &= (I + GK)^{-1} \\ R &= K(I + GK)^{-1} \\ T &= GK(I + GK)^{-1} \end{aligned} \quad (30)$$

where  $S$  is called of sensibility,  $T$  complementary sensitivity function, and  $R$  doesn't have any name. The function cost of mixed sensibility is named like, because it punishes  $S$ ,  $R$  and  $T$  at the same time; it can also be said as project requirement. The transfer function from  $w$  to  $z_1$  is the weighted sensitivity function,  $W_1 S$ , which characterizes the performance objective of good tracking; the transfer function from  $w$  to  $z_2$  is the complementary sensitivity function  $T$ , whose minimization ensures low control gains at high frequencies, and the transfer function from  $w$  to  $z_3$  is  $KS$ , which measures the control effort. It is also used to impose the constraints on the control input; for example, the saturation limits.

#### 4. NUMERICAL SIMULATIONS

The purpose of the simulation is to demonstrate the performance of the developed model and controller algorithm. The simulation model is realized in the MATLAB. The Law of Control  $\tau$  is a simple proportional derivative (PD), where the gains  $K_1$  and  $K_2$  are determined through simulations (Souza, 2006) represented by

$$\tau = -K_1\theta - K_2\dot{\theta} \quad (31)$$

The initials conditions used here are  $\theta=0.001$  rad and  $\dot{\theta}=0$  rad/s. The values considered for the physical parameters in the numerical simulation are presented in Tab. 1.

Table 1. Physical parameters

Parameter	Description	Value
$J_0$	Moments of inertia of the rigid body of the satellite	720 Kg.m <sup>2</sup>
$J_p$	Moment of inertia of the panel	40 Kg.m <sup>2</sup>
$K$	Constant elastic of the panels	320 Kg.rad <sup>2</sup> /s <sup>2</sup>
$K_d$	Dissipation constant	0,48 Kg.rad <sup>2</sup> /s
$L$	Length of the panel	2 m
$m$	Mass of the satellite	20 kg

The weighting matrices considered here are:

$$Q = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} ; \quad R = |1| \quad (32)$$

The structure of the weighting matrices of Kalman Filter (LQG) is given by

$$W = |w.w'| \quad V = \begin{bmatrix} v.v' & 0 & 0 & 0 \\ 0 & v.v' & 0 & 0 \\ 0 & 0 & v.v' & 0 \\ 0 & 0 & 0 & v.v' \end{bmatrix} \quad (33)$$

For the Eq. (33), the values will be analyzed through cases presented in the Tab. 2.

Table 2. Weights of the Kalman Filter

	w	v
Case 1	0.0001	0.1
Case 2	0.001	10 <sup>-6</sup>
Case 3	10 <sup>-6</sup>	0.01

The procedure of the project of  $H_\infty$  is different to other control projects; the difference is the use of weighting functions  $W_1$ ,  $W_2$  and  $W_3$ . Where,  $W_2=0$  and the others are given by

$$W_1^{-1} = \gamma^{-1} * \frac{0.1(1+s/100)^2}{(1+s/5000)^2} * I_{2 \times 2} \quad (34)$$

$$W_3^{-1} = \frac{2000}{s} * I_{2 \times 2} \quad (35)$$

$W_1$  corrects the error sign  $e$ ,  $W_2$  corrects the control sign "u", and  $W_3$  corrects the exit of the plant  $y$ ; and  $\gamma$  is a parameter obtained through successive attempts.

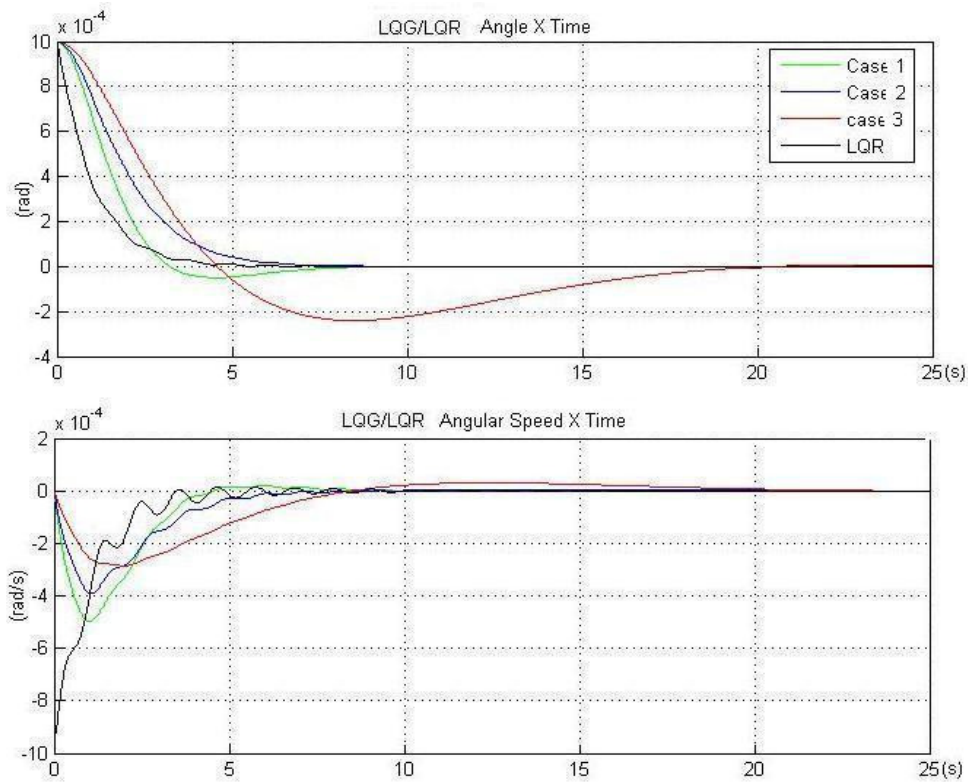


Figure 5. LQG/LQR angle and angular speed

The performance of LQR and LQG are represented in the Fig. 5. Making a comparison of the cases of LQG amongst themselves, the worst result is obtained by the case 3, with a time of larger stabilization than 20 seconds, and the overshoot very distant of the origin. However, comparing LQG with LQR sees himself an improvement in the overshoot, so much for angle as for the angular speed. That is due to the filter of Kalman, used as an estimator of the flexible states. However, nor all the curves of LQG reach the time of stabilization of LQR and the overshoot of the curves meet moved away of the origin.

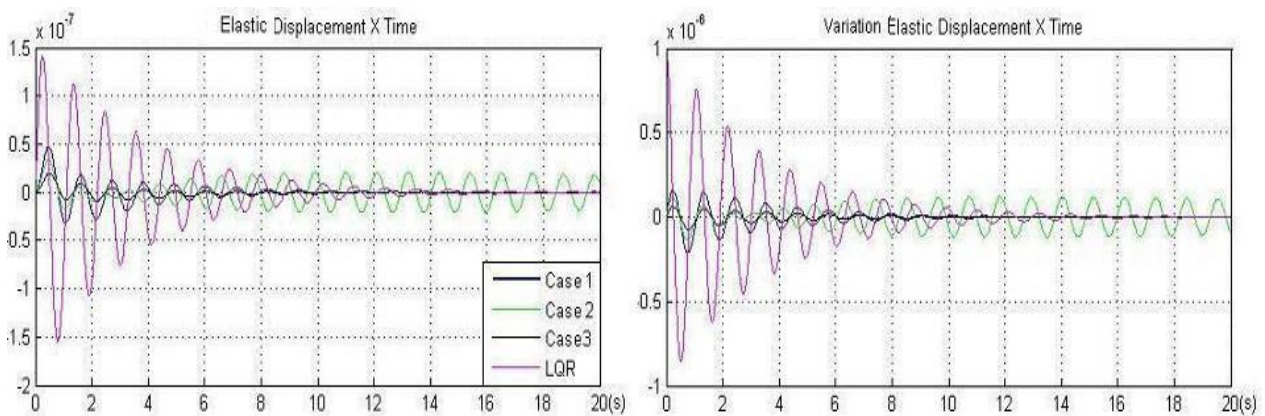


Figure 6. LQG/LQR vibration of the panels

In the results of the Fig. 6, it is had that the use of the filter of Kalman as an estimator of the flexible states of the satellite, supplied to the system a great result in comparison with the result of LQR. Except for the case 2, that it doesn't converge in the time of 20 seconds, the other cases demonstrate a time of superior stabilization to LQR. More especially in the case 3, this demonstrated a time of stabilization of about 12 seconds. The displacements of overshoot were also smaller, mainly in the case 3, compared to LQR and to the other cases.



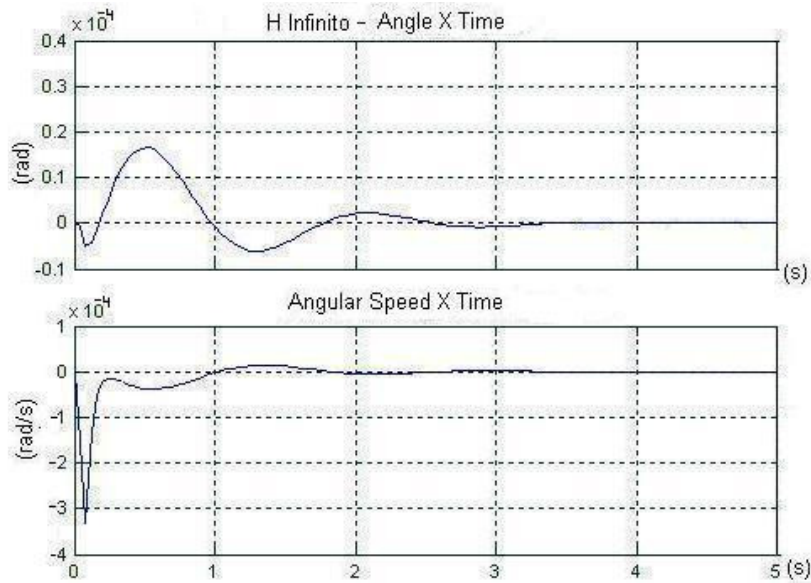


Figure 7.  $H_\infty$  angle and angular speed

The performance of  $H_\infty$  is observed in Fig. 7. Both graphs have existence of overshoot, in which they could commit the system; however the time of stabilization of both was of approximately 3.5 seconds. In other words, in spite of the existence of overshoots, the control of the system, in a long time was reached. In comparison with the results of LQR and LQG the time of stabilization with  $H_\infty$  is smaller. In LQR and LQG the time of stabilization was five to six seconds.

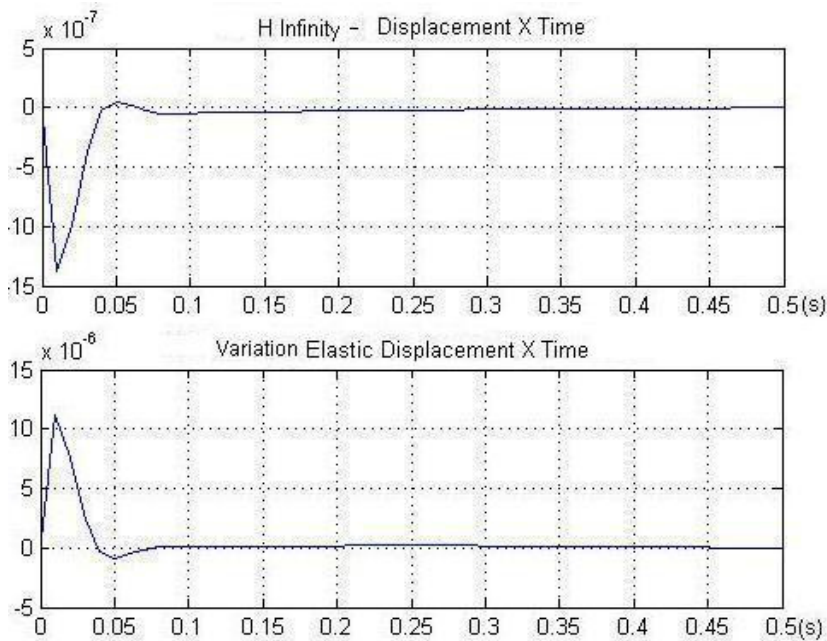


Figure 8.  $H_\infty$  vibration of the panels

In the Fig. 8 the behavior of the vibration of the panels is presented. The displacement of overshoot is of the order of  $10^{-7}$ , in other words, very small as in LQR and LQG. The time of stabilization in the first graph is about 0.5 seconds, and for the second one is about 0.45 seconds.

That demonstrates that the control  $H$  possesses a better performance for angle and angular speed, as well as for the vibration of the panels.

## 5. CONCLUSIONS

The observations and conclusions on the control methods applied in this paper are presented.

**LQR method:** The LQR method is not adapted in cases that all of the states are not available and when the rejection of the noises and the robustness of the system have great importance in the project. The adjustment of matrixes weights  $Q$  and  $R$  of the project for the method try and error, does not offer a precision of the best values than can exist. The choice of matrixes should be made in agreement with the needs and/or priorities of the project. Therefore, the LQR method is more appropriate for systems that have project models reasonably exact and ideal sensors/actuators; and in the preliminary apprenticeship of the project of the control laws.

**LQG method:** The LQG method overcomes some inconveniences of the LQR method. LQG demonstrates a better result when the Kalman filter is used as an estimator of the flexible states, occurrence observed in the levels of vibration of the panels. However, the action of the system declines in comparison with LQR for the angle and the angular speed due to the presence of Kalman filter. Once again, the choice of matrixes ( $Q$ ,  $R$ ,  $W$ , and  $V$ ) should be done in agreement with the needs and/or priorities of the project. Therefore, comparing LQG to LQR, LQG is more realistic. Because it can be used when nor all of the states are available and when the system presents noises.

**$H_\infty$  method:** The  $H_\infty$  method design technique is one of the most advanced techniques available today for designing robust controllers. One great advantage with this technique is it allows the designer to tackle the most general form of control architecture wherein explicit accounting of uncertainties, disturbances, actuator/sensor noises, actuator constraints, and performance measures can be accomplished. The systematic is very different to the methods LQR and LQG. However, a great disadvantage is the experience and necessary abilities to design the form of the weighting functions, the increased plant and the value of  $\gamma$ . The success of the method depends, basically, of the correct choice of the functions transfer weights.

## 6. ACKNOWLEDGEMENTS

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## 8. RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.